Professor Per Jensen, Ph.D.
Physical and Theoretical Chemistry
School of Mathematics and Natural Sciences

Fakultät für Mathematik und Naturwissenschaften
Fachgruppe Chemie & Biologie

BERGISCHE UNIVERSITÄT WUPPERTAL

Fundamentals of Molecular Symmetry ***Exercises***

Week 1/11

1.1 The multiplication table of a group can be viewed as a matrix in which the position ij (in the ith row and jth column) is occupied by the product $A_i\,A_j$, where A_i is the element at the left end of the ith row and A_j is that at the top of the jth column. Use the group axioms to show that in given row of this matrix, no two elements can be equal, so that each group element must occur once and only once in the row. Show that an analogous result applies to each column of the matrix.

1.2 Since, as shown in Problem 1.1, each group element occurs once and only once in the rows and columns of the group multiplication table, we can construct such multiplication tables in an abstract manner using a technique similar to that of the Japanese game SUDOKU. Try this out by considering a group of four elements

$$\boldsymbol{G} = \{E, A, B, C\}\,.$$

E is the neutral element.

a) Assume that $A^2 \neq E$, $B^2 \neq E$ and $C^2 \neq E$. That is, no element except E is its own inverse element. Construct the possible multiplication tables of the group.

b) Assume that among the three elements A, B, C, only $C = C^{-1}$. That is, $A^2 \neq E$, $B^2 \neq E$ and $C^2 = E$. Determine the possible multiplication tables of the group.

c) Determine the possible multiplication tables of a four-element group for which $B^2 = C^2 = E$, but $A^2 \neq E$.

d) Determine the possible multiplication tables of a four-element group for which $A^2 = B^2 = C^2 = E$.

e) How many fundamentally different four-member groups are there?

1.3 Construct the multiplication table for the so-called cyclic group with five elements:

$$\boldsymbol{G} = \left\{E, C_5, C_5^2, C_5^3, C_5^4\right\}.$$

This multiplication table is the only one allowed for a five-element group.